LEBANESE AMERICAN UNIVERSITY
Division of Computer Science and Mathematics

Discrete Structures I

Exam I

Fall 2012 (October 31)

Name: Solutions **ID:** _____

Question Number	Grade
1. 10%	
2. 10%	
3. 8%	
4. 7%	
5. 12%	
6. 12%	
7. 13%	
8. 13%	
9. 15%	
Total	

1. (10%) Given that the sets A, B and C are all countable, show that their union is also countable.

Let $D = A \cup B \cup C$.

$A = \{a_1, a_2, \dots\}$ $B = \{b_1, b_2, \dots\}$ $C = \{c_1, c_2, \dots\}$

Let $D = \{d_1, d_2, \dots\}$ so that.

$a_1 = d_1$ $a_2 = d_4$ $a_3 = d_7$

$b_1 = d_2$ $b_2 = d_5$ $b_3 = d_8$ etc...

$c_1 = d_3$ $c_2 = d_6$ $c_3 = d_9$

$\therefore$ D is countable.

2. (10%) If you know that x is rational and y is irrational, show whether or not $2x + \frac{3}{2}y$ is rational **Specify the proof type you use.**

$z = 2x + \frac{3}{2}y$ is irrational

Spse not [proof by contradiction] $\Rightarrow$.

Spse $z = \frac{m}{n}$

$y = \frac{2}{3}\left[z - 2x\right] = \frac{2}{3}\left[\frac{m}{n} - 2x\right]$

but since x is rational $\Rightarrow$ $x = \frac{a}{b}$, for some

$a, b \in \mathbb{Z}^* \Rightarrow$ $y = \frac{2}{3}\left[\frac{m}{n} - \frac{2a}{b}\right]$ also rational,

a contradiction, since y is irrational

3. (8%) Write a compound proposition that is true when EXACTLY one of the three propositions p, q, or r are true.

We want: $[r \to p \vee q] : F$ or $[\sim r \vee p \vee q] : F$ or $\sim[\sim r \vee p \vee q]$

$\Rightarrow \boxed{r \wedge \sim q \wedge \sim p}$

4. (7%) A logician told her son: "If you don't finish your dinner, you will not get to stay up to watch TV." He finished his dinner and then was sent to bed. Discuss.

No dinner (p) $\longrightarrow$ No TV (q).

or. TV $\longrightarrow$ dinner.

He finished his dinner $\Rightarrow \sim p$. or $p : F$.

So the statement is. $p : F \Longrightarrow$ either q or $\sim q$ are allowed.

$F \to T$ or $F \to F$: both ok.

5. (12%) Show that the following are equivalent. Don't use truth tables: $(p \to q) \wedge (p \to r)$ and $(p \to q \wedge r)$

$(p \to q) \wedge (p \to r) : (\sim p \vee q) \wedge (\sim p \vee r)$

$= \sim p \vee (q \wedge r)$ } same $\Rightarrow$

$p \to q \wedge r : \quad \sim p \vee (q \wedge r)$ }

equivalent

6. (12%) Given the proposition: "if we finish work on time, then if the weather permits, we will go for ice cream"

$f \longrightarrow (w \longrightarrow i)$

(a) Represent this compound proposition with symbols.

where f : we finish work on time.
w: weather permits
i: go for ice cream.

or $f \wedge w \longrightarrow i$

(b) Write its contrapositive. $\neg i \longrightarrow \neg(f \wedge w)$

or $\neg i \longrightarrow \neg f \vee \neg w$

(c) Write its negation We finished on time, and the weather permitted, yet we did not go for ice cream.

7. (13%) Consider the statement: "The difference between a real number and itself is zero". Express this statement symbolically using predicates, quantifiers, logical connectives etc.. .

$x, y \in$ Real numbers.

$P(x, y) : x - y = 0.$

$\forall x \; P(x, x)$

or $\forall x \left(P(x, y) \longrightarrow x = y\right)$ only.

8. (13%) Given the proposition $\forall x \exists y \, (F(x) \wedge P(x)) \rightarrow M(x, y)$

Make up your own English statement that agrees with the above. Make sure you specify the domains of x and y.

9. (15%) Show using induction that $8 \mid 5^{n+1} + 2 * (3^n) + 1$ for all $n = 1, 2, 3, \ldots$

1) Basic Step
$n=1$ ⟹ $8 \mid 5^2 + 6 + 1$ $\quad 8 \mid 32$ ✓

2) Ind. Step

Assume $P(k)$, show $P(k+1)$.

Assume $5^{k+1} + 2 \cdot 3^k + 1 = 8c$, $\Rightarrow \boxed{5^{k+1} = 8c - 2 \cdot 3^n - 1}$

show $5^{k+2} + 2 * 3^{n+1} + 1 \stackrel{?}{=} 8d$.

$5 \cdot 5^{k+1} + 2 * 3^{n+1} + 1 \stackrel{?}{=} 8d$.

$5(8c - 2 \cdot 3^n - 1) + 2 * 3^{n+1} + 1$

$40c - 10 \cdot 3^n - 5 + 6 * 3^n + 1$

$= 40c - 4 * 3^n - 4$

$= 4(10c - 3^n - 1) = 4 * \text{even}$

$= 8d.$

(10c: even; 3^n: even; $10c - 3^n$: even; $10c - 3^n - 1$: even)